Strength Prediction of Short Fibre Reinforced Polymers

This contribution is focused on the strength prediction of injection moulded short fibre reinforced polymer parts. Two strategies for the application of different strength prediction criteria are developed and applied. It is shown, that the selection of a certain criteria has a significant influence on the simulation results, the number of material tests necessary for calibration and the engineering expenses. To be able to verify the computed results and to evaluate the pros and cons of the described strategies and applied criteria, the results are compared to experimental tests.

Vorhersage der Festigkeit von kurzfaserverstärkter Kunststoffen

In der vorliegenden Arbeit werden Dimensionierungskriterien fokussiert, die es ermöglichen die Festigkeit kurzfaserverstärkter, polymerer Bauteile vorherzusagen. Zwei Strategien zur Anwendung unterschiedlicher Kriterien werden entwickelt und angewendet. Es wird gezeigt, dass die Auswahl eines Kriteriums zur Vorhersage der Festigkeit einen wesentlich Einfluss auf die Simulationsgüte, auf die Anzahl der benötigten Versuche zur Kalibrierung und auf die Entwicklungskosten hat. Zur Verifizierung der berechneten Ergebnisse und zur Evaluierung der Vor- und Nachteile der Dimensionierungskriterien und Strategien werden diese mit experimentellen Ergebnissen verglichen.
Strength Prediction of Short Fibre Reinforced Polymers

J.-M. Kaiser, M. Stommel

In this contribution two strategies to predict the strength of injection moulded short fibre reinforced polymer parts are investigated and compared. The strategies are based on different strength criteria, which are commonly applied in practical engineering design (e.g. Tsai-Hill) and are combined with a two-step mean-field homogenization approach. The approach provides the opportunity to account for the heterogeneous microstructure of a polymer composite, caused by the commonly non-unidirectional fibre distribution due to the injection moulding process. In a two-step homogenization approach a representative volume (domain) of the polymer composite with its heterogeneous fibre distribution is considered as a composition of weighted, unidirectional sub-domains. The chosen modelling approach allows the application of criteria to predict the strength after both homogenization steps. This leads to two different strength prediction strategies. The selection of a certain criteria in combination with the selected level of strength prediction influences the simulation results and the number of material tests necessary. Thus, these two aspects are directly linked to engineering expenses and they are seen as the necessary focus of this contribution. Finally, to be able to verify the computed results and to evaluate the advantages and disadvantages of the described strategies and applied criteria, the results are compared to experimental test data.
1 INTRODUCTION

It is well known that composite materials show complex fracture behaviour, which is commonly a combination of fibre pullout, matrix breakage and fibre breakage. In the engineering aspect not only the capability of a model to account for different fracture modes must be considered. Rather the selected model must be applicable in practical engineering design and a calibration should be possible with a minimum effort of experimental tests. This is the advantage of the so-called non-differentiating criteria, which are commonly based on stress or strain states and which can be calibrated with standardized tests. An overview over such criteria is given in [1]. These strength predicting criteria can be differentiated into two categories. Category (1) contains so-called linear criteria, which cannot take into account interactions between the stresses or strains. In contrast the so-called non-linear criteria of category (2) offer the possibility to take into account interactions of acting stresses or strains. These criteria are also known as “quadric failure criteria” [2]. Criteria of both categories were successfully applied by Lopez et al. [3] in a comparative study to laminated composites. In this contribution criteria of both categories are investigated, evaluated and applied to short fibre reinforced polymers.

In the last decade several authors have successfully predicted progressive damage, failure and strength of polymer composite materials based on mean-field homogenization models (e.g. [1] - [15]). The micromechanical models are based on Eshelbys' fundamental solution for spheroid inclusions embedded in an infinite matrix [14] and the work of Mori et al. [15] and Tandon et al. [16]. They enable the evaluation of mean stresses and strains in the matrix and the fibre. These mean values are the starting point failure modelling and strength prediction. The micromechanical homogenization model used in this contribution is implemented according to [19 - 22] and allows the calculation of the rate independent elasto-plastic behaviour of composites with arbitrarily oriented inclusions. The approach provides the opportunity to account for a heterogeneous microstructure, which is caused by the commonly non-unidirectional fibre distribution due to the injection moulding process. To take into account such a microstructure a representative volume (domain) is considered as a composition of weighted, unidirectional sub-domains. The homogenization requires a two step procedure. In a first step an incremental Mori-Tanaka homogenization scheme is applied to the unidirectional sub-domains. In a second step a Voigt model is used to finally compute the mechanical composite behaviour of the domain. This procedure is seen as an effective way to successfully predict a composites mechanical behaviour and hence, make it highly useful for practical applications [23].

Based on this two-step mean-field homogenization approach two strength-prediction strategies are introduced. The first strategy includes strength prediction based on the unidirectional sub-domain level, while the second one in-
Strength prediction on the commonly non-unidirectional domain level. Thus, the two strategies allow the application of different strength criteria, which vary essentially in the number of material tests necessary for calibration. The simulation results of the different strategies are compared to experimental test data and a cost benefit analysis is presented, which compares and evaluates the chosen strategy and the selected criteria.

2. STRENGTH PREDICTION CRITERIA

Short fibre reinforced polymer composites show complex fracture behaviour, which is commonly a combination of fibre pullout, matrix breakage and fibre breakage. A typical fracture surface is shown in Figure 1.

![Figure 1: Scanning electron microscope picture of a fracture surface](image)

In this contribution the application of non-differentiating criteria to predict strength are combined with a two-step mean-field homogenization approach. A strength criterion can be written as:

\[
f(\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{13}, \tau_{12}, F_1, \ldots, F_n)
\]

where \(\sigma_1, \ldots, \tau_{12}\) are the stresses and \(F_1, \ldots, F_n\) are the required strength parameters. To be able to consider strains \(\varepsilon_1, \varepsilon_2, \varepsilon_3, \gamma_{23}, \gamma_{13}, \gamma_{12}\) the stresses have to be replaced in (1). Failure occurs, when a critical value is reached for \(f\):

\[
\begin{align*}
  f < 1 : & \text{ no failure} \\
  f = 1 : & \text{ failure limit} \\
  f > 1 : & \text{ failure}
\end{align*}
\]

Four criteria are selected, two of both categories. The first category includes linear criteria and interactions between acting stresses or strains are not considered. For a general anisotropic material they are defined by:
\[
F \sigma_i < 1 \quad (3)
\]

where

\[
F_i^T = (F_1, F_2, F_3, F_4, F_5, F_6) \quad (4)
\]

The second group of failure criteria includes non-linear criteria and interactions between acting stresses and strains are considered. In the general anisotropic case such a criterion is defined by:

\[
F \sigma_i + F_i \sigma < 1 \quad (5)
\]

and:

\[
F_{ij} = \begin{bmatrix}
F_{11} & F_{12} & F_{13} & F_{14} & F_{15} & F_{16} \\
F_{21} & F_{22} & F_{23} & F_{24} & F_{25} & F_{26} \\
F_{31} & F_{32} & F_{33} & F_{34} & F_{35} & F_{36} \\
F_{41} & F_{42} & F_{43} & F_{44} & F_{45} & F_{46} \\
F_{51} & F_{52} & F_{53} & F_{54} & F_{55} & F_{56} \\
F_{61} & F_{62} & F_{63} & F_{64} & F_{65} & F_{66}
\end{bmatrix} \quad (6)
\]

The applied strength prediction criteria must consider a microstructure as shown in Figure 2, which represents a general anisotropic reference volume of an injection moulded polymer composite.

Figure 2: Real microstructure (scanning electron microscope picture) and its model representation

The application of a linear criterion, as defined by equation (3), would require 6 experimental tests. Furthermore, the application of a non-linear criterion, as defined by equation (5), would require 27 experimental tests. Hence, an application of strength criteria at this level is out of reach from the engineering point of view. Therefore, in a first step the considered microstructure, Figure 2, is divided into weighted, transversal isotropic sub-domains, Figure 3.

Figure 3: Definition of transversal isotropic sub-domains
The experimental effort is significantly reduced by considering a transversal isotropic sub-domain for strength-prediction. Figure 4 shows possible experimental tests for a transversal isotropic material.

**Figure 4: Schematic illustration of possible tests**

In Figure 4 $X_i, Y_i, R, X_c, Y_c, Q$ are the strength parameters, wherein $i$ indicates tension tests and $c$ compression tests. $R$ and $Q$ are the required shear test strength parameters. Based on these results, the following criteria can be defined and applied:

a) maximum stress criteria

\[-X_c < \sigma_1 < X_i, \quad |\tau_{23}| < Q\]
\[-Y_c < \sigma_2 < Y_i, \quad |\tau_{13}| < R\]
\[-Y_c < \sigma_3 < Y_i, \quad |\tau_{12}| < R\]

No failure occurs when all conditions in (7) are fulfilled.

b) maximum strain criteria

\[-X_{ce} < \varepsilon_1 < X_{ic}, \quad |\gamma_{23}| < Q_e\]
\[-Y_{ce} < \varepsilon_2 < Y_{ic}, \quad |\gamma_{13}| < R_e\]
\[-Y_{ce} < \varepsilon_3 < Y_{ic}, \quad |\gamma_{12}| < R_e\]

Again, no failure occurs when all conditions in (8) are fulfilled. With the assumption of transversal isotropic material behaviour equation (6) is simplified and the well known Tsai-Hill criterion can be defined:
A further simplification is possible if the structures are made of isotropic materials. This case is referred to as the von-Mises criterion and equation (6) reduces to only one value:

d) von-Mises
\[
F_{ij} = F_{11}
\] (10)

A detailed derivation for the calculation of required the strength parameters in (9) and (10) can be found in [1]. A practical conduction of the required tests for a Tsai-Hill, maximum stress and maximum strain criteria is possible if the reinforced material is transversal isotropic. The application of the von-Mises failure criteria requires an isotropic material behaviour. The total number of tests required for the calibration of the selected criteria in this contribution and the required parameters are summarized in Table 1.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Required Test Parameters</th>
<th>Number of Tests</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>von-Mises</td>
<td>Xc</td>
<td>1</td>
<td>isotropic material behaviour required</td>
</tr>
<tr>
<td>Max. Stress</td>
<td>Xc, Xb, Yc, Yb, Q, R</td>
<td>4</td>
<td>transversal isotropic material behaviour required; no differentiation between tension and pressure</td>
</tr>
<tr>
<td>Max. Strain</td>
<td>X(<em>{ct}), X(</em>{ct}), Y(<em>{ct}), Y(</em>{ct}), Q(<em>{ct}), R(</em>{ct})</td>
<td>4</td>
<td>transversal isotropic material behaviour required; no differentiation between tension and pressure</td>
</tr>
<tr>
<td>Tsai-Hill</td>
<td>Xb, Yb, R, Q = f (F(<em>{22}), F(</em>{23}))</td>
<td>3</td>
<td>transversal isotropic material behaviour required; no differentiation between tension and pressure</td>
</tr>
</tbody>
</table>

Table 1: Summary of the applied criteria
3. **STRENGTH PREDICTION MODELING**

The embedding of the criteria defined in section 2 into a micromechanical model is described in the following subsections 3.1 and 3.2. A two-step homogenization approach is chosen, which results into two different strength prediction strategies. The following notations are used. Dots and colons are used to indicate tensor products contracted over one and two indices. Tensor products are defined by \( \otimes \).

\[
x \cdot y = x_{ij} y_{ij}, \quad x : y = x_{ij} y_{ji}, \quad (x \otimes y)_{ijkl} = x_{ij} y_{kl}.
\]

Boldface symbols donate tensors; the order is indicated by the context. Einstein’s summation convention over repeated indices is used unless otherwise indicated (2). The averages of any kind are always indicated by \( \langle \cdot \rangle \).

\[
x_{ik} y_{kj} \equiv \sum_{k=1}^{3} x_{ik} y_{kj}.
\]

### 3.1 First-Level Strength Prediction Modelling

Each sub-domain has a volume \( v \) and consists of a matrix phase with a volume fraction \( v_0 \) and an inclusion phase with a volume fraction \( v_1 = 1 - v_0 \). In the first-level modelling a sub-domains is considered, which is transversal isotropic. This is shown in combination with the assumed load case in Figure 5.

Figure 5: **Sub-domain level modelling**

Therein, \( \langle \epsilon \rangle \) is the average applied composite strain and \( \langle \epsilon \rangle_{v_0} \) and \( \langle \epsilon \rangle_{v_1} \) are the average strains in fibre and matrix, respectively. With the help of Eshelby’s fundamental solution [16] one can define a relationship for the average strains in the constituents:

\[
\langle \epsilon \rangle_{v_1} = E : \langle \epsilon \rangle_{v_0}
\]

and one for the average strains between the constituents and the composite [24]:
\[
\langle \varepsilon \rangle_{v_0} = [v_1E + (1 - v_1)I]^{-1} : \langle \varepsilon \rangle \tag{14}
\]

\[
\langle \varepsilon \rangle_{v_i} = E[v_1E + (1 - v_1)I]^{-1} : \langle \varepsilon \rangle. \tag{15}
\]

Therein, \( I \) designates the fourth order symmetric identity tensor. In equation (13), (14) and (15) \( E \) is the strain concentration tensor, which is defined by [24], [25]:

\[
E = \{I + S(I, s_0) : [s_0]^{-1} : s_i - I\}^{-1} \tag{16}
\]

Therein, \( s_i \) is the stiffness tensor of the reinforcement and \( s_0 \) is the stiffness tensor of the matrix material. \( S \) is the so-called Eshelby tensor, which depends on the geometry \( I \) and on \( s_0 \). It can be calculated according to Mura [26]. Assuming linear material behaviour, a stiffness tensor \( s \) can now be expressed as:

\[
\langle \sigma \rangle = s : \langle \varepsilon \rangle. \tag{17}
\]

and the composite stiffness tensor \( s \) is given by [18] :

\[
s = [v_1s_i : E + (1 - v_1)s_0] : [v_1E + (1 - v_1)I]^{-1}. \tag{18}
\]

Due to this result, it is possible to calculate the homogenized stiffness tensor for a composite with arbitrarily dimensioned spheroid inclusion on the unidirectional sub-domain level, Figure 6.

**Figure 6: Illustration of the first-level homogenization procedure**

Equation (18) completes the first-level mean-filed homogenization modelling. At this level, a non-linear Tsai-Hill and a linear maximum stress and strain criterion is applied to the composite material of a sub-domain. The mean composite stresses and strains can be calculated. If the strength limit is exceeded, the failed sub-domain is no longer considered (Figure 7) and expression (19) is valid.

**Figure 7: Illustration of the first-level strength prediction**

\[
\langle \sigma \rangle_{i} = \frac{(1 - 3v_1)E + (1 - v_1)I}{(1 - 2v_1)E + (1 - v_1)I} : \langle \varepsilon \rangle_{i}. \tag{19}
\]
\[ s_{i=n} = 0 \]  (19)

### 3.2 Second-Level Strength Prediction Modelling

In the next step the micromechanical approach must be extended to capture orientation distributions. Therefore an orientation averaging procedure must be applied. A two step procedure is chosen to predict the overall mechanical properties of such a composite [21,27]. In a first step the stiffness tensor for a unidirectional composite on the sub-domain level is calculated, see subsection 3.1 equation (18). In a second step the overall stiffness tensor of the domain is calculated with the help of an orientation averaging procedure [28]:

\[ \langle s \rangle = \int \langle s(p) \rangle \psi(p) dp \]  (20)

In (9) \( p \) is a unit vector, which is associated with a fibre orientation and \( \psi(p) \) is the fibre distribution function, which can be calculated as proposed by [28]. \( \langle s(p) \rangle_{i=n} \) represents one sub-domain. The second level homogenization step is shown in Figure 8.

![Illustration of the second-level homogenization procedure](image)

The Voigt model assumption completes the so called second-level mean-field homogenization since it is assumed that:

\[ \langle \varepsilon \rangle = \langle \varepsilon_i \rangle, i = 1...N \]  (21)

It is pointed out here that the modelling approach was already successfully applied by Pierard et al. [29] and succeeded in further publications [30], however without embedding and introducing strength prediction strategies. After the second-level homogenization a commonly anisotropic domain is considered, Figure 9. A domain itself consists of several unidirectional, transversal isotropic subdomains. When the strength limit is reached at this level equation (22) is valid and the whole domain fails.
As derived in subsection 3.1 the calibration of an anisotropic failure criterion is normally out of reach. However, the Mori-Tanaka homogenization scheme allows the calculation of the mean strains in the constituents and thus, the mean strains and stresses in an isotropic matrix as shown in equation (14). With the assumption that the strength limit is based on matrix failure a von-Mises criterion and a maximum stress and strain criterion are applicable as a second-level strength prediction.

In a last step the previously introduced two-step homogenization approach is extended to matrix plasticity, while the reinforcement is assumed to be linear elastic and isotropic. For each sub-domain a tangent moduli $s^{tg}$ has to be defined, which fulfills:

$$\langle \sigma \rangle_i = s^{tg} : \langle \dot{\varepsilon} \rangle_i, i = 1...N$$  \hspace{1cm} (23)

More detailed information about the numerical implementation and about the applied return mapping algorithm can for example be found in [31]. In literature, there exist several possibilities about how to define a tangent moduli $s^{tg}$, which is necessary to consider elasto-plastic behavior of a constituent. In this contribution a von-Mises elasto-plasticity model is chosen. However, the implementation also allows the consideration of other elasto-plasticity models. The von-Mises elasto-plasticity yield criterion reads:

$$f = \sigma_{eq} - R(p) - \sigma_y \leq 0$$  \hspace{1cm} (24)

where $\sigma_y$ is the initial yield stress, $p$ the accumulated plastic strain, $R(p)$ the hardening function, which defines the hardening stress, and $\sigma_{eq}$ the von-Mises measure of the applied stress. Detailed information about the implemented elasto-plastic formulation can be found in [15], [31-33] and [34]. The applied criterion and the level of application are summarized in Table 2.
Criteria | First-Level Application (Composite Failure) | Second-Level Application (Matrix Failure)  
---|---|---
von-Mises | – | X  
Max. Stress | X | X  
Max. Strain | X | X  
Tsai-Hill | X | –  

*Table 2: Selected criteria and their level of application*

4. EXPERIMENTAL RESULTS

The material used in the experimental part is Grivory HTV-3H1 by EMS with a weight fraction of 30 % glass fibres. The material properties are given in Table 3. Tests on the matrix material, a polyamide PA6, were not conducted since these values are normally also not available. The required mechanical properties were determined in a reverse engineering approach with the software Hyperstudy 10 from Altair. The results are also given in Table 3. The experimental tension strength parameters of the composite required for the calibration of the failure models were determined on tensile test bars (DIN EN ISO 527, Type 1BA). They were milled out of injection moulded plates under three different angles, 0°, 45° and 90°. The 45° samples are not mandatory but useful for verification reasons. The required shear tests were conducted on Iosipescu shear samples, which were also produced by injection moulding and milling. Again 0° and 90° samples were tested. The results for the tests are summarized in Table 4.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus Fibre</td>
<td>7300 MPa</td>
</tr>
<tr>
<td>Poisson’s Ratio Fibre</td>
<td>0.22</td>
</tr>
<tr>
<td>Weight Fraction of Fibres</td>
<td>30 %</td>
</tr>
<tr>
<td>Young’s Modulus Matrix</td>
<td>3650 MPa</td>
</tr>
<tr>
<td>Poisson’s Ratio Matrix</td>
<td>0.34</td>
</tr>
<tr>
<td>Aspect Ratio of Fibres</td>
<td>20</td>
</tr>
</tbody>
</table>
| Plastic Hardening Law         | \( f(\sigma, p) = \sigma_{\text{eq}} - \sigma_y - R(p) \)  
|                               | \( R(p) = \left[ 15p + 32\left(1 - e^{-350p}\right) \right] \) MPa  
|                               | \( \sigma_y = 30 \) MPa |

*Table 3: Considered material properties*
Table 4: Summary of the experimental test results

<table>
<thead>
<tr>
<th>Test</th>
<th>Experimental Results for Stress</th>
<th>Experimental Results for Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension 0°</td>
<td>151 MPa</td>
<td>1.64 %</td>
</tr>
<tr>
<td>Tension 45°</td>
<td>105 MPa</td>
<td>2.24 %</td>
</tr>
<tr>
<td>Tension 90°</td>
<td>98 MPa</td>
<td>2.52 %</td>
</tr>
<tr>
<td>Shear 0°</td>
<td>108 MPa</td>
<td>1.90 %</td>
</tr>
<tr>
<td>Shear 90°</td>
<td>123 MPa</td>
<td>2.32 %</td>
</tr>
</tbody>
</table>

The reference part, which was chosen to be able to evaluate the simulation results, is shown in Figure 10 together with the load case and the experimental result. Failure occurs at 4.2 mm displacement and 640 N. Details about the injection locations are given in Figure 11 and its simulated counterpart is shown in Figure 12. In Figure 12 the first principle fibre orientation tensor value is displayed. The three principal values of the fibre orientation tensor together with the corresponding coordinate system are required for each element to evaluate equation (20).
Besides the experimental results "virtual" tension tests were conducted to determine the mean strength parameters of the matrix material, e.g. von-Mises stress, at the point of failure. These parameters cannot be determined in experimental test, but are necessary to be able to apply the failure criteria of the second level strength prediction (matrix failure). Taking into consideration equation (14) the calculation of the mean matrix strain or stress is possible. Therefore, the missing matrix failure parameters can be determined in a FEM simulation of the composite material by defining the required mean values of the ma-
trix as an additional output variable. The virtual tensile test results are shown in Figure 13 and are listed in Table 5.

![Virtual tensile test results](image-url)

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Numerical Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>von-Mises</td>
<td>85 MPa</td>
</tr>
<tr>
<td>Max. Stress</td>
<td>84 MPa</td>
</tr>
<tr>
<td>Max. Strain</td>
<td>2.9 %</td>
</tr>
</tbody>
</table>

Table 5: Summary of the virtual test results

5. FINITE-ELEMENT SIMULATION RESULTS

In subsection 3.1 and 3.2 two strength-prediction strategies are presented. They are now applied to the chosen reference structural part in the following subsections 5.1 and 5.2.

5.1 First-Level Strength Prediction

At this level composite failure is considered. The strategy presented in subsection 3.1 is combined with different criteria at this level, which results in a gradual degradation of the overall domain stiffness due to the failure of sub-domains.
However, as it can be seen in Figure 14, this effect is only slightly visible. The simulation results of this level are summarized in Table 6.

![Simulation results for the first-level strength prediction](image)

**Figure 134: Simulation results for the first-level strength prediction**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Numerical Results</th>
<th>Deviation from Experimental Results [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tsai-Hill</td>
<td>4.16 mm</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>655 MPa</td>
<td>2.34</td>
</tr>
<tr>
<td>Max. Stress</td>
<td>3.95 mm</td>
<td>5.95</td>
</tr>
<tr>
<td></td>
<td>620 MPa</td>
<td>3.13</td>
</tr>
<tr>
<td>Max. Strain</td>
<td>3.95 mm</td>
<td>5.95</td>
</tr>
<tr>
<td></td>
<td>620 MPa</td>
<td>3.13</td>
</tr>
</tbody>
</table>

**Table 6: Summary of simulation results of the first-level strength prediction**

### 5.2 Second-Level Strength Prediction

At this level only matrix failure is considered and hence, only the virtual determined matrix results can be used to predict the failure of the composite. If the critical matrix stress or strain value is reached in any domain, then the calculation is aborted. The results are shown in Figure 15 and are summarized in Table 7.
Figure 145: Simulation results for the second-level strength prediction

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Numerical Results</th>
<th>Deviation from Experimental Results [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>von-Mises</td>
<td>4.10 mm</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td>650 N</td>
<td>1.56</td>
</tr>
<tr>
<td>Max. Stress</td>
<td>3.30</td>
<td>21.43</td>
</tr>
<tr>
<td></td>
<td>525 N</td>
<td>17.97</td>
</tr>
<tr>
<td>Max. Strain</td>
<td>4.40</td>
<td>4.76</td>
</tr>
<tr>
<td></td>
<td>700 N</td>
<td>9.38</td>
</tr>
</tbody>
</table>

Table 7: Summary of simulation results for the first-level strength prediction

6. CONCLUSION

In this contribution a two-step homogenization approach is combined with two different strength prediction strategies. After both homogenization steps strength criteria are applied and results are compared. The main focus of this procedure is seen in the evaluation of the area of conflict between a chosen strength criteria, its performance and the number of experimental test necessary to calibrate the chosen criteria. These three aspects are summarized in combination with a proposal for rating in Table 8.
It has to be mentioned that the second-level strategy can be used for part strength estimation even without any experimental test conduction. For the determination of the mechanical properties of the matrix material and for the conduction of the virtual tests, which are necessary to determine the required strength parameters, at least one experimental stress-strain curve with a corresponding failure stress and strain has to be known. On one hand these data can be determined in tests with a 0°, 45° or 90° sample. On the other hand in databases like CAMPUS (www.campusplastics.com) experimental results are commonly available for a 0° test specimen. Hence, a substitution of the experimental test is possible. The authors clarify at this point that this proposed method is, and can only be, a first estimation and if possible experimental test under known conditions should be used for calibration.

None the less the second-level strategy is capable of predicting the strength with a reasonable accuracy (2.38 %, von-Mises criterion, 4.76 %, max. strain criterion) in this study. It is well known that the usage of the von-Mises equivalent stress should be avoided for polymer parts. In a next step more sophisticated plasticity models have to be integrated into the proposed two-step homogenization approach. Another focus of future work is seen in the necessary differentiation between tension and pressure, especially in the context of the first-level strength prediction. This will enable the use of the Tsai-Wu failure criteria for example. However, this also leads to a revision of the ratings in Table 8, since this extension also increases the number of test necessary to calibrate the applied criteria.

Table 8: Summarization of the results

<table>
<thead>
<tr>
<th>Level</th>
<th>Criteria</th>
<th>Virtual Experiment</th>
<th>Real Experiments</th>
<th>Deviation [%] (displacement)</th>
<th>Cost-Benefit Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Tsai-Hill</td>
<td>0</td>
<td>3</td>
<td>0.95</td>
<td>+ +</td>
</tr>
<tr>
<td>1st</td>
<td>Max. Stress</td>
<td>0</td>
<td>4</td>
<td>5.95</td>
<td>-</td>
</tr>
<tr>
<td>1st</td>
<td>Max. Strain</td>
<td>0</td>
<td>4</td>
<td>5.95</td>
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<td>1</td>
<td>4.76</td>
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+ + very good; + good; - fair; - - poor
References


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</tr>
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<td>------------------</td>
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<td>-------</td>
</tr>
<tr>
<td>[28]</td>
<td>Cintra, J.S.; Tucker, C.L.</td>
<td>Orthotropic Closure Approximations for Flow-Induced Fiber Orientation</td>
</tr>
<tr>
<td>[29]</td>
<td>Pierard, O.; Friebel, C.; Doghri, I.</td>
<td>Mean-Field Homogenization of Multi-Phase Thermo-Elastic Composites: A General Framework and its Validation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Composites Science and Technology 64 (2004) p.1587-1603</td>
</tr>
<tr>
<td>[30]</td>
<td>Pierard, O.; Doghri, I.</td>
<td>An Enhanced Affine Formulation and the Corresponding Numerical Algorithms for the Mean-Filed Homogenization of Elasto-Viscoplastic Composites</td>
</tr>
<tr>
<td>[31]</td>
<td>Doghri, I.</td>
<td>Mechanics of Deformable Solids</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Springer Verlag Berlin, 2000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ISTE Publishing London, 2007</td>
</tr>
<tr>
<td>[33]</td>
<td>Pierard, O.</td>
<td>Micromechanics of inclusion-reinforced composites in elasto-plasticity and elasto-viscoplasticity modeling and computation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Doctoral Thesis Louvain, 2006</td>
</tr>
<tr>
<td>[34]</td>
<td>Ponte Castañda, P.</td>
<td>Exact second-order estimates for the effective mechanical properties of nonlinear composite materials</td>
</tr>
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Keywords:
Homogenization, strength prediction, short fibre reinforced polymer composites

Stichworte:
Homogenisierung, Festigkeitsvorhersage, kurzfaserverstärkte Kunststoffe

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